November 2012 subject reports

MATHEMATICS SL

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 19	20 - 38	39 - 53	54 - 64	65 - 76	77 - 87	88 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 13	14 - 19	20 - 23	24 - 28	29 - 33	34 - 40

The range and suitability of the work submitted

All schools submitted tasks drawn from the current set provided by the IB.

Candidate performance against each criterion

Criterion A

The use of various calculator notations and the lack of an appropriate approximately equals sign continues to be a problem in Criterion A.

Criterion B

While most candidates provided good, clear communication there persist cases where candidates used a “question – answer” style, added information as appendices or labelled graphs poorly. Teachers and candidates should note that background information in the form of lessons on the material are not expected and can hinder the overall quality of communication.

Criterion C

In a Type I task it is crucial that an analysis be presented based upon sufficient data produced from the initial mathematical strategy. Regression methods are inappropriate as no analysis is presented to demonstrate understanding of the process. Once a conjecture is made further values must be used to validate the proposed general statement, and the results must be compared to the results obtained from the original pattern. It is not appropriate to use the same values that were used to develop the conjecture.

In a Type II task, models must be developed analytically to score higher than C2. If an incorrect model is produced through analytic methods and all further models are achieved through regression techniques then only C2 is possible. A common approach was to use regression techniques to find a model that fits, then to use analysis to "develop" the model function. This is inappropriate as it does not allow the candidate to demonstrate their own understanding of how certain functions might fit certain graphs. The analysis must come first and the regression work can be used afterwards to confirm the model or compare for best fit. Comments on how well the data fit the model function were often superficial. Statements like "the graph fits the data well" are not enough to gain C4. Candidates must also apply their own model to a further set of data and discuss the fit in order to attain level C5. Modifications to the model to ensure a better fit are awarded under D5.

Criterion D

In Type I tasks most candidates were able to attain the lower levels of the criteria. However, the quality of exploration of the scope and limitations varied widely. Those candidates with good creative thinking skills would consider a variety of possible values and then demonstrate whether or not they were valid. Very few were able to provide a suitable informal explanation.

In Type II tasks the biggest area of concern is that many candidates get caught up in the pure mathematical treatment of the data and forget to interpret the model in the context of the task. Where they do remember, the interpretations are often superficial. Teachers should take note that progress beyond D2 requires interpretation in context and that the higher levels cannot be attained unless significant and critical interpretation is provided. A little bit of research on the topic can be immensely helpful in this regard. Level D5 not only requires significant interpretation of results, but also that the original model be modified to fit a new set of data. Simply creating a new model from scratch is insufficient. Accuracy also needs better consideration. A model is usually, by nature, inaccurate to some degree. Candidates should consider how changes in the accuracy of parameters affect or don't affect the quality of the model.

Criterion E

Teachers have different views on how technology can enhance the presentation of work. It is less a question of what technology was used than how the technology helped the reader understand the solution. To include simple graphs printed from a calculator or computer is not as valuable as constructing the graphs so that comparisons between relations or functions can be easily seen. Spreadsheets that easily allow for exploration of the effect of large *n* in a Type I task are another good resource. Above all teachers should think carefully about their own expectations for the use of technology, and then make this clear to the moderator in the background information provided with the sample.

Criterion F

This criterion was well understood by the great majority of teachers. Levels F0 and F2 were awarded rarely and appropriately.

Recommendations and guidance for the teaching of future candidates

Teachers must take the time to explain the criteria to their students and clarify any confusion as to the meaning of the criteria. Assistance in this is available on the Online Curriculum Centre, especially through the document "Internal assessment criteria and additional notes".

It would be very helpful for candidates to have a practice task of each type prior to the tasks that are intended for submission to the IB. Older IB published tasks are perfect for this.

The concepts and processes of conjecture and modelling should be taught in class using examples that resemble the kinds of tasks that candidates will explore. Concepts such as validation of a conjecture and interpretation of a model in context are not well understood.

Teachers should model the effective use of technology in their lessons so that candidates can appreciate the potential of such technology. There are many good graphing software packages available for schools to purchase or for individual use on a trial basis.

Comments from teachers are highly encouraged as these help the moderator to understand why certain levels were awarded. Teachers should feel free to mark up the candidates' work with both positive and constructive comments. The candidates may see their work after marking so that they can take note of what worked well and what didn't. The teacher must promptly collect the intact and unaltered work for safe-keeping.

Further comments

Coordinators should ensure that feedback and subject reports are read by teachers so that common repetitive issues of concern are addressed. Many schools seem unsure as to what documents should be provided with the sample sent to the moderator. As these requirements can change from time to time it is essential that teachers and coordinators work together to ensure that the appropriate supporting documents are completed properly and included. Aside from any official forms, samples must include copies of tasks used (including those published by the IB), solution keys and/or marking schemes for each task submitted, and background information on previous work done that relates to the tasks.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 18	19 - 37	38 - 49	50 - 59	60 - 70	71 - 80	81 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Recognizing the sign of a trigonometric ratio for an angle not in the first quadrant
- Finding an axial intercept for a vector equation in three dimensions
- Using the discriminant
- Vector geometry
- Using the chain rule to find a derivative
- Reasoning skills

The areas of the programme and examination in which candidates appeared well prepared

In general, most candidates were able to make a good attempt on each question, with very few questions left blank. Candidates were often successful in approaching straightforward situations with correct methods. Candidates showed good preparation and knowledge in the following areas:

- working with matrices
- discrete random variables and expected value
- differentiation and integration of polynomials
- using the quotient rule to find a derivative
- finding information from a cumulative frequency curve
- magnitudes of vectors

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 - Matrices

The large majority of candidates answered this question successfully. There were only a small number of candidates who seemed to have never worked with matrices before. Occasionally a candidate would incorrectly approach part (b) by finding an inverse of matrix ***A***.

Question 2 - Discrete Probability

Overall, this question was very well done. A few candidates left this question blank, or used methods which would indicate they were unfamiliar with discrete random variables. In part (b), there were a good number of candidates who set up their work correctly, but then had trouble adding or multiplying decimals without a calculator. A common type of error for these candidates was $5(0.4)=0.2$.

Question 3 - Integrals and Volume

Many candidates answered both parts of this question correctly. In part (b), a large number of successful candidates did not seem to notice the link between parts (a) and (b), and duplicated the work they had already done in part (a). Also in part (b), a good number of candidates squared $(x-4)$ in their integral, rather than squaring $\sqrt{x-4}$, which of course prevented them from noting the link between the two parts and obtaining the correct answer.

Question 4 - Derivatives and Gradient

A majority of candidates answered part (a) correctly, and a good number earned full marks on both parts of this question. In part (b), some common errors included setting the derivative equal to zero, or substituting 3 for x in their derivative. There were also a few candidates who incorrectly tried to work with $f(x)$, rather than $f'(x)$, in part (b).

Question 5 - Trigonometric Identities

While many candidates correctly approached the problem using Pythagoras in part (a), very few recognized that the cosine of an angle in the second quadrant is negative. Many were able to earn follow-through marks in subsequent parts of the question. A common algebraic error in part (a) was for candidates to write $\sqrt{1-m^2}=1-m$. In part (c), many candidates failed to use the double-angle identity. Many incorrectly assumed that because $\sin 100° = m$, then $\sin 200° = 2m$. In addition, some candidates did not seem to understand what writing an expression "in terms of *m*" meant.

Question 6 - Vector Equation of a Line in Three Dimensions

In part (a), the majority of candidates correctly recognized the equation that contains the position and direction vectors of a line. However, we saw a large number of candidates who continue to write their equations using "$L =$", rather than the mathematically correct "$\boldsymbol{r} =$" or "$\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$". $\boldsymbol{r}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ represent vectors, whereas *L* is simply the name of the line. For part (b), very few candidates recognized that a general point on the *x*-axis will be given by the vector $\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$. Common errors included candidates setting their equation equal to $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, or $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, or even just the number 0.

Question 7 - Discriminant

The majority of candidates who attempted to answer this question recognized the need to use the discriminant, however very few were able to answer the question successfully. The majority of candidates did not recognize that the quadratic equation must first be set equal to zero. In addition, many candidates simply set their discriminant equal to zero, instead of setting it greater than zero. Even many of the strongest candidates, who obtained the correct numerical values for k, were unable to write their final answers as a correct interval.

This question is a good example of candidates who reach for familiar methods, without really thinking about what the question is asking them to find. There were many candidates who attempted to solve for x using the quadratic formula or factoring, even though the question did not ask them to solve for x.

Question 8 - Cumulative Frequency and Quartiles

Many candidates answered this question completely correctly, earning full marks in all parts of the question. In parts (a) and (b), there were some who gave the frequency values on the *y*-axis, rather than the wages on the *x*-axis, as their quartiles and inter–quartile range.

For part (c), the majority of candidates seemed to understand what was required, though there were a few who used an extreme value such as 700, rather than the median value.

In part (d), some candidates simply answered 65, which is the number of workers earning $500 or less, rather than finding the number of workers who earned more than $500. It was interesting to note that quite a few candidates gave their final answer as 14, rather than 15.

Question 9 - Vectors in Two Dimensions

Part (a) was answered correctly by nearly every candidate.

In part (b), the candidates who realized that the vectors must be perpendicular were successful using the scalar product to find p. Incorrect approaches included using magnitudes, or creating vector equations of lines for the sides and setting them equal to each other. In addition, there were a good number of candidates who worked backwards, using the given value of 3 for p to find the coordinates of point D. Candidates who work backwards on a "show that" question will earn no marks.

Part (c) was more difficult for candidates, and was left blank by some. Some candidates found $\overrightarrow{AC}$ rather than $\overrightarrow{OC}$, as required. Many candidates recognized that the opposite sides of the rectangle must be equal, but did not consider the directions of the vectors for those sides. There were also a good number of candidates who mislabelled the vertices of their rectangles, which led to them working with a rectangle ABDC, rather than ABCD.

The majority of candidates who attempted part (d) were successful in multiplying the magnitudes of the sides. Unfortunately, there were some who set up their solutions correctly, then had arithmetic errors in their working.

Question 10 - Derivatives and Integrals

In part (a), most candidates recognized the need to apply the quotient rule to find the derivative, and many were successful in earning full marks here.

In part (b), many candidates struggled with the chain rule, or did not realize the chain rule was necessary to find the derivative. Again, some candidates attempted to work backward from the given answer, which is not allowed in a "show that" question. A few clever candidates simplified the situation by applying properties of logarithms before finding their derivative.

For part (c), many candidates recognized the need to integrate the function, and that their integral would equal $\ln 4$. However, many did not recognize that the integral of h is g. Those candidates who made this link between the parts (b) and (c) often carried on correctly to find the value of k, with a few candidates having errors in working with logarithms.

Recommendations and guidance for the teaching of future candidates

Candidates should be given the chance to work with past exams for practice. This is a good way for teachers and students to discuss what is required by the different command terms, such as "show that", "find", or "write down". Working with past questions also helps candidates practice their problem-solving skills by working through questions that are being asked in different ways.

Candidates should also be encouraged to read each question carefully and consider what a question is asking them to do before they begin their work. Too often, candidates jump in with a formula or a familiar process which may be unnecessary or incorrect in the given situation. This was evident in question 7, where many candidates used the quadratic formula, rather than the discriminant, or used the discriminant without considering the nature of the roots.

Candidates can also look for clues within the given information, especially in questions with multiple parts. Question 3 is a good example, where candidates were able to use their answer from part (a) to quickly find the answer to part (b). Another example of this is in question 10(c), where many candidates spent valuable time trying unsuccessfully to integrate an unfamiliar function, when the answer they needed was given to them in part (b) of the question.

As always, candidates should be encouraged to show all their working in a neat, organized manner which is easy to follow and not randomly scattered all over the page. If a mistake is made, it is best to simply draw an "X" or a line through any unwanted working.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 50	51 - 59	60 - 69	70 - 78	79 - 90

The areas of the programme and examination which appeared difficult for the candidates

Candidates in this session had difficulties in the following areas of the programme:

- Correct use of parenthesis when expanding a binomial
- Understanding and use of the command terms “sketch” and “show that”
- Sketching graphs carefully to show important points and using the correct domain
- Normal distribution especially finding the value of a standardized variable
- Conditional probability and the meaning of independent events
- Finding maximum velocity from displacement function
- Area between two curves
- Use of a graphic display calculator (GDC) to evaluate definite integrals

The areas of the programme and examination in which candidates appeared well prepared

For students who were well prepared, there was ample opportunity to demonstrate a high level of knowledge and understanding on this paper. The following areas of the programme were handled well by most students.

- Arithmetic series
- Sketching the graph of a function using the graphing calculator and finding intercepts.
- Quadratic functions
- Circle geometry and trigonometry
- Area of a sector and arc length
- Cosine rule
- Matrices and their inverses

- Finding gradients and the equations of a normal
- Finding parameters of trigonometric functions
- Use of Venn diagrams
- Simple probability

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 Arithmetic sequences and series

Most candidates performed well on this question. A few were confused between the term number and the value of a term.

Question 2 Matrices and their inverses

Most candidates answered part (a) without difficulties, understanding the requirements of the GDC to find the inverse of a 3x3 matrix. In part (b), some candidates either multiplied on the wrong side or wrote their matrix on the wrong side but multiplied correctly on their GDC. Those who tried to solve the system analytically struggled in vain, leading to algebraic errors.

Question 3 Equation of a Normal

This question was generally done well. Most candidates did not use their GDC in part (b), resulting in a variety of careless errors occasionally arising either in differentiating or substituting. There were some candidates who did not know the relationship between gradients of perpendicular lines while others found the equation of the tangent rather than the normal in part (c).

Question 4 Binomial Expansion

This question proved challenging for many students. Most candidates recognized the need to expand a binomial but many executed this task incorrectly by selecting the wrong term, omitting brackets, or ignoring the binomial coefficient. Other candidates did not recognize that there were **two** values for p when solving their quadratic equation.

Question 5 Trigonometric Functions

Part (a) (i) was well answered in general. There were more difficulties in finding the correct value of the parameter c. Finding the correct value of b in part (b) also proved difficult as many did not realize the period was equal to 8. Most candidates could handle part (c) without difficulties using their GDC or working with the symmetry of the curve although follow through from errors in part (b) was often not awarded because candidates failed to show any working by writing down the equations they entered into their GDC.

Question 6 Normal Distribution

A standard question for which well-prepared candidates frequently earned all eight marks. Common errors included the use of percentages rather than z-values and the inability to find the negative z-value. Others had correct equations but were not able to use their GDC to solve them and ultimately made errors in their algebra.

Question 7 Graph sketching, displacement and velocity

Most candidates sketched an approximately correct shape for the displacement of a particle in the given domain, but many lost marks for carelessness in graphing the local extrema or the right endpoint. In part (b), most candidates knew to differentiate displacement to find velocity, but few knew how to then find the maximum. Occasionally, a candidate would give the time value of the maximum. Others attempted to incorrectly set the first derivative equal to zero and solve analytically rather than take the maximum value from the graph of the velocity function.

Question 8 Arc length, area of a sector, area of a triangle

Candidates generally handled the cosine rule, sectors and arcs well, but some candidates incorrectly treated triangle AOB as a right-angled triangle. A surprising number of candidates changed all angles to degrees and worked with those, often leading to errors in accuracy. In part (c), some candidates misread the question and used 2.4 as the size of angle AOC while others rounded prematurely leading to the inaccurate answer of 48. In either case, marks were lost. Part (d) proved to be straightforward and candidates were able to obtain full FT marks from errors made in previous parts. Most candidates had a suitable strategy for part (e) and knew to work with a whole number of cans of paint.

Question 9 Quadratic functions, transformations and area between two curves

A good number of students provided a clear sketch of the quadratic function within the given domain. Some lost marks as they did not clearly indicate the approximate positions of the most important points of the parabola either by labelling or providing a suitable scale. There were few difficulties in part (b) but in part (c), candidates often used an insufficient number of steps to show the required result or had difficulty setting out their work logically. Part (d) was generally done well though many candidates gave at least one answer to fewer than three significant figures, potentially resulting in more lost marks. In part (e), many candidates were unable to connect the points of intersection found in part (d) with the limits of integration. Mistakes were also made here either using a GDC incorrectly or not subtracting the correct functions. Other candidates tried to divide the region into four areas and made obvious errors in the process. Very few candidates subtracted $f(x)$ from $g(x)$ to get a simple function before integrating and there were numerous, fruitless analytical attempts to find the required integral.

Question 10 Probability

Parts (a) and (b) were generally done well although some candidates left answers as decimals rather than the required percentages. In part (c) (i), candidates failed to find the intersection of the events as, in general, they multiplied probabilities, assuming the events were independent or they incorrectly attempted to use the union formula. Independence in (c) (ii) caused difficulty with some candidates attempting to use the conditions for mutually exclusive events while others assumed the events were independent in part (i) and then found $\mathrm{P}(G \cap S)$ by multiplying $\mathrm{P}(S|G) \times \mathrm{P}(G)$. Part (d) proved quite challenging as a great majority could only find the probability of being a boy. Those who did attempt it, and successfully connected the problem with conditional probability, often had difficulties in reaching the correct final answer.

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to follow all instructions, including rubrics, and to give their answers in required forms, if specified in the question. There are many who still prematurely round values leading to inaccurate final answers. Candidates should be encouraged to work with more than three significant figures and should be aware that answers left to one significant figure will obtain no marks while those left to two or incorrectly rounded to three significant figures may also lose marks.

Candidates should be taught not simply to transcribe graphs from their GDC without considering their intrinsic knowledge of key features and behaviours of functions. The command term "sketch" is still not well understood although its definition is clearly stated in the guide. Candidates should be encouraged to use the appropriate GDC tools to find key features of graphs rather than estimate them using “trace” functions.

It is evident that many candidates are often unable to decide if an algebraic approach or a GDC approach is required. Teachers should ensure that their students feel confident about using their calculators as the primary method of finding solutions on Paper 2.

There seems to be an increasing tendency for some candidates to incorrectly label the sub-parts of a question. This makes marking very difficult and teachers and students must be aware that if the students present their work in the wrong sub-part, they will possibly not earn marks as the examiner does not know precisely which part of the question the candidate is trying to answer. Teachers should encourage students to label each part of their answer exactly as given in the question and to emphasize the need to present clearly communicated work.

In "show that" questions, stress that students must approach this problem from the beginning and strive to reach the conclusion indicated using an appropriate number of steps, even if some steps are rather obvious. Some candidates continue to substitute in values and work backwards, thinking that this is the evidence that is required.

Teachers must also stress to students the importance of checking the mode of their calculators to determine if they are using radians or degrees. In particular, when they are asked to sketch the graph of a trigonometric function, teachers must emphasize the students to set their calculator in radians.

Communication when using GDC still needs more emphasis; “found using GDC” is insufficient working. Including sketches and equations entered into the GDC will ensure follow through marks can be awarded if errors are made in previous parts.

Graphical approaches to derivatives and graphical relationships between a function and its derivatives should be emphasized particularly for Paper 2. Analytical relationships are primarily examined on Paper 1.

Design the course in such a way as to provide adequate time for students to develop conceptual understanding in conjunction with good technique and timely use of a graphic display calculator. Encourage understanding through reading and communicating appropriate mathematical language. Expose students to more mathematics set in both familiar and unfamiliar contexts particularly in the areas of trigonometry and calculus.